Title: Word Problems Involving Linear Systems in Two Variables
Class: Math 100 or Math 107
Author: Sharareh Masooman
Instructions to tutor: Read instructions under "Activity" and follow all steps for each problem exactly as given.
Keywords/Tags: word problems, applications, real-life, linear systems, mixture problems

Objective: Algebra is a language that can be used to represent real-life problems and answer questions. We need to learn to analyze word problems and translate them into algebraic language that would help us get to solutions of application problems.

Activity: Follow the given steps to translate the word problems into linear systems. (This worksheet emphasizes writing systems - you should already know how to solve them.) Do at least one example with a tutor first, then try the rest of the examples on your own. Take them to your tutor to make sure you're doing them correctly. Keep track of the steps listed in each example and use them to solve the word problems at the end of this activity.

Example 1. The crew for the spaceship ZETA must be chosen from aliens of 2 different species: Romulans and Webbans. Romulans have 2 hands and Webbans have 8 hands. If there are exactly 30 crew members, and the crew must have a total of exactly 84 hands, then how many aliens of each species must be chosen?

1. Write the "question" here: (this one's done for you) How many aliens of each species must be chosen?
2. How many things are you being asked to find? One? two? more than two?
3. Which of the following would be appropriate labels for the unknowns in this problem? (only one choice)
(a) Let $x$ and $y$ represent the size of the crew
(b) Let x represent the number of Romulans and let y represent the number of Webbans in the crew
(c) Let x and y represent the number of hands in the crew
(d) Let x represent Romulans and let y represent Webbans

Hopefully you picked (b) above. Make sure you understand why the other choices are not correct.
4. We know that there are exactly 30 crew members.

Translate "total number of crew members $=\mathbf{3 0}$ " into an algebraic equation:

We also know that there are exactly 84 hands.
Translate "total number of hands $=\mathbf{8 4}$ " into an algebraic equation:
(look up how many hands each alien species have)
5. Write the two equations below and solve the system.

6. Use your solution to the system to answer the question you wrote in \#1.
$\qquad$ Romulans and $\qquad$ Webbans.

Example 2. A 9000-seat amphitheater has tickets for sale at $\$ 35$ and $\$ 62$. How many tickets should be sold at each price for a sellout performance to generate a total revenue of $\mathbf{\$ 3 8 2 , 5 0 0}$ ?

1. Write the "question" here: How many
2. How many things are you being asked to find? One two more than two
3. Which of the following would be appropriate labels for the unknowns in this problem? (one choice only)
(a) Let x and y represent the tickets
(b) Let x represent the $\$ 35$-tickets and let y represent the $\$ 62$-tickets
(c) Let $x$ represent the number of $\$ 35$-tickets and let $y$ represent the number of $\$ 62$-tickets
(d) Let x represent the price of one kind of tickets and let y represent the price of other kind of tickets

Hopefully you picked (c) above. Make sure you understand why the other choices are not correct.
4. The amphitheater has 9000 seats so a sellout performance means "total number of tickets sold $=9000$ ". Translate this statement into an algebraic equation:

Now translate the statement "total revenue $=382,500$ " into an algebraic equation. (Keep in mind that money made from selling each kind of ticket is the price of that ticket times how many of those tickets were sold)
5. Write the two equations below and solve the system.

6. Use your solution to the system to answer the question you wrote in \#1. (In English!)

Should sell $\qquad$
Example 3. Fred invests $\$ 3000$ more in an account that pays $\mathbf{4 \%}$ simple interest than he invests in an account that pays $3 \%$ simple interest. If the total interest earned for the first year is $\$ 610$, how much did Fred invest in each account?

1. Write the "question" here: How much
2. How many things are you being asked to find? One two more than two
3. Which of the following would be appropriate labels for the unknowns in this problem?
(a) Let x represent amount invested in each account
(b) Let x and y represent amount invested in each account
(c) Let x represent the $3 \%$ account and let y represent the $4 \%$ account
(d) Let x represent amount invested in the $3 \%$ account and let y represent amount invested in the $4 \%$ account

Hopefully you picked (d). Make sure you understand why the other choices are not correct.
4. We know that Fred invested $\$ 3000$ more in the $4 \%$ account than the $3 \%$ account, i.e. "amount invested in the $4 \%$ account $=3000$ more than amount invested in the $3 \%$ account.
Translate this statement into an algebraic equation:

Now translate the statement "total interest $=610$ " into an algebraic equation. (Keep in mind that interest comes from interest rate times amount invested)
5. Write the two equations below and solve the system.

6. Use your solution to the system to answer the question you wrote in \#1. (In English!)

Fred invested $\qquad$
Let's try a similar problem:
Example 4. Fran invested $\$ 700$ in two funds that pay $6 \%$ and $5.5 \%$ annual interest. Fran's combined annual interest is $\mathbf{\$ 4 0 . 5 0}$. How much of the $\$ 700$ was invested in each fund?

1. Write the "question" here: $\qquad$
2. How many things are you being asked to find? One two more than two
3. Label the variables. (Refer to Example 3)

Let x represent:
Let y represent:
4. We know that Fran invested a total of $\$ 700$, i.e. "total amount invested $=700$ ".

Translate this statement into an algebraic equation:

Now translate the statement "total interest $=40.50$ " into an algebraic equation. (Keep in mind that interest comes from interest rate times amount invested)
5. Write the two equations below and solve the system.

6. Use your solution to the system to answer the question you wrote in \#1. (In English!)

Fran invested $\qquad$

## Mixture Problems

Example 5. How many gallons of a $\mathbf{2 0 \%}$ antifreeze solution and a $\mathbf{1 0 \%}$ antifreeze solution must be mixed to obtain 40 gallons of a $\mathbf{1 6 \%}$ antifreeze solution?

1. Write the "question" here:
2. How many things are you being asked to find? One two more than two
3. Label the variables. Make sure you are clear and precise.

Let x represent: number of gallons of $\qquad$
Let y represent: $\qquad$
4. We want to have 40 gallons of the new ( $16 \%$ ) solution. So the total number of gallons must be 40 . (A table might help with this type of problem to organize all the information.)
Translate the statement "total number of gallons of mixtures $=40$ " into an algebraic equation:

We also know that the new solution should be a $16 \%$ solution, i.e. $16 \%$ of the 40 gallons which is (0.16)(40) must be pure antifreeze.

Now translate this statement into an algebraic equation:
"gallons of pure antifreeze in $20 \%$ solution + gallons of pure antifreeze in $10 \%$ solution = gallons of pure antifreeze in the $16 \%$ solution"
5. Write the two equations below and solve the system.

6. Use your solution to the system to answer the question you wrote in \#1. (In English!)

## After you go over the previous problems with a tutor, try the following on your own, then check with a tutor to make sure you did them correctly.

For each of the following problems label the variables, set up a system, solve it and interpret the result.

1. The sum of two numbers is 24 . One is 6 less than twice the other. Find the two numbers.
2. Two hundred eighty-two people attended a recent performance of Cinderella. Adult tickets sold for $\$ 5$ and children's tickets sold for $\$ 3$ each. Find the number of adults and the number of children that attended the play if the total revenue was $\$ 1046$.
3. Shana invests some money in an account that earns $5 \%$ simple interest and three times that amount in an account that earns $7 \%$ simple interest. If the total interest is $\$ 390$ at the end of one year, find the amount invested in each account.
4. A bulk store has two types of mixed nuts: One is $60 \%$ peanuts, and the other is $35 \%$ peanuts. How many pounds of each type should be mixed to get 8 pounds of a mixture that is $50 \%$ peanuts?

For tutor use: Please check the appropriate box.
Student has completed worksheet but may need further assistance. Recommend a follow-up with instructor.
$\square$ Student has mastered topic.

